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METHODS OF MATHEMATICAL MODELING OF THE AREA OF CARBONATION OF WOODEN BEAMS WITH LINING OF FIRE PROTECTIVE PLYWOOD

The analysis of existing methods of mathematical modeling of estimation of fire resistance of building constructions remains an actual subject for researches as, there are errors of researches which should vary with experimental researches. The analysis makes it possible to choose a more advanced integrated model of fire description. According to the results of the work, the following was established:

We analyzed the existing mathematical methods for modeling heat and mass transfer processes in the creation of heat furnaces to determine the limit of fire resistance of wooden beams, taking into account possible research errors.

Based on the obtained experimental data due to the fire tests of samples-fragments of wooden beams lined with fire-retardant plywood.

Based on the solution of the differential equation of nonstationary thermal conductivity of Fourier's law and taking into account the finite difference method, a calculation model was created to determine the charring zone of wooden beams lined with fire-retardant plywood.

The complexity of the problem in the case of transition to differentiated zonal studies, which optimize the calculation of experimental data for further theoretical generalization, is not reduced, because the process zones located in space have a complex volumetric nature, even in the simplest cases of flame burning.

Key words: *fire-retardant plywood, mathematical modeling, fire temperature, fire propagation, field methods.*

Formulation of the problem. Integrated, zone and field models are now used to predict fire hazards [1, 3]. Integrated models make it possible to obtain a forecast of the average values of the parameters of the state of the environment in the room for any moment of fire development. In zone models, the entire space of the fire is divided into characteristic spatial zones and determine the average values of the parameters of the state of the environment in these zones for any moment of fire development. Field or differential models of fire make it possible to predict the spatio-temporal distribution of temperatures and velocities of the gaseous medium in the room, the concentrations of environmental components, pressure and densities at any point in the room. When determining the spatio-temporal distribution of temperature and velocity of the gaseous medium to calculate the limit of fire resistance.

Analysis of recent achievements and publications. Existing models of fires, in the study of which a number of scientists have invested: S.V. Puzach, IF Astakhanov, A.M. Ryzhov, YU.A. Nightmares - which are aimed at specific solutions to fire safety problems, reflect individual cases of combustion of substances and materials in the room of certain configurations.

CFAST models take into account the transferred masses and heat by horizontal and vertical air flows.

Computational Fluid Dynamics (CFD) allows you to calculate the flow of gases and liquids, as well as the distribution of temperature and pressure and other values at any point in the simulated room, but has high requirements for computer hardware [2]. NIST FDS is a CFD model of fire in which turbulent motion is described by the laws of conservation of full momentum and total energy using the Large Eddy Simulation (LES) model. First of all, the tasks of smoke transfer and heat transfer are solved. The fire propagation model based on Markov finite chains is stochastic as opposed to deterministic approaches to modeling fire development.

With a large field of research, the issues of mathematical modeling of the charring zone of wooden beams lined with fire-retardant plywood remain unsolved.

In [2] by means of computer gas-hydrodynamics Flow Vision. This paper demonstrates the results of numerical simulation of a number of computer configurations of the installation for testing building structures. From the results of this work it became known that in this software package you can also see the uneven distribution of temperatures on the heating surface of structures during tests for fire resistance.

Highlighting previously unsolved parts of the general problem to which the article is devoted. According to previous studies [3–5], methods and tools have errors, so it is advisable to conduct additional research and establish the possibility of modeling heat and mass transfer to identify the probability of reproduction of test conditions for fire resistance of building structures.

Existing scientific works have not tested the possibility of modeling installations for testing the fire resistance of building structures.

Thus, we determined the temperature distributions in the cross section of the fragment of the wooden beam, which was tested using the recommendations containing the relevant standard for the calculation methods for assessing the fire resistance of wooden building structures.

Problem statement and its solution. To solve this goal, a number of tasks were identified:

- to analyze the possibility of a software package for modeling thermodynamic processes;
- develop a construction algorithm;
- create an installation for testing the fire resistance of building structures;
- analyze the results.

Presentation of the main material of the study with a full justification of the results.

Existing mathematical models are described by nonstationary thermal conductivity. Mathematical model of heat transfer in load-bearing elements of beams during their fire tests. The Computational Fluid Dynamics (CFD) technique [6] allows to calculate the flows of gases and liquids, as well as the distribution of temperature and pressure and other quantities. This technique is based on the formulation and solution of boundary and initial problems for systems of differential equations that express the laws of conservation of mass, moments and energy. Such problems are solved using numerical methods on a computer (for example, the finite element method). This technique solves a very wide range of problems related to the physics of liquid and gas flows. The most well-known software products that use the CFD technique:

- ANSYS CFX;
- FLUENT;
- STAR – CD;
- FLOW VISION;
- FEATFLOW.

To study the temperature distributions in the cross section of the load-bearing beams in fire tests, a calculation technique based on the solution of the heat equation having the form [7-9] was used:

$$Cv(T)\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda(T)\frac{\partial T}{\partial y}\right), \quad (1)$$

where *T* - temperature, ºC;

t - hour, c;

Cv (T) - specific volume heat capacity, depending on temperature, J / (m^3/°C);

λ(T) - temperature-dependent thermal conductivity, W / (m^3/°C).

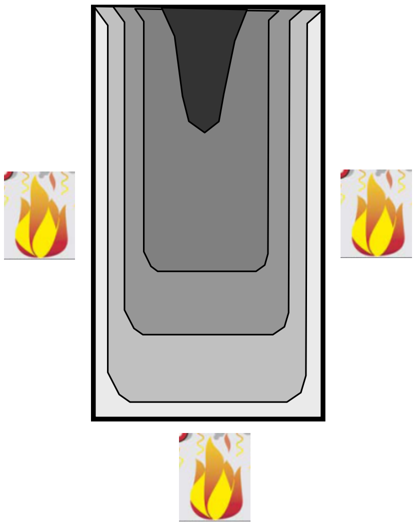

Fig. 1. Geometry and boundary conditions for thermal calculation of reinforced concrete beams.

A rectangular section of a wooden beam with three-sided heating is considered, the scheme of which is shown in Fig. 1.

To solve the thermal problem, the boundary conditions (GU) of the third kind are used:

$$-\lambda(T)\frac{\partial T}{\partial r} = \alpha(T_P - T_W), \tag{2}$$

where α - heat transfer coefficient, W /(м^2·°C);
T_P, T_W - respectively the temperature of the fire environment and the surface of the beam, °C;
r - flow spatial coordinate.

The ambient temperature is determined by the standard temperature curve of the fire by the formula:

$$T_p(t) = 345 \cdot \lg\left(\frac{2t}{15} + 1\right) + T_0, \tag{3}$$

where: T_0 - initial temperature of the medium, °C; $T_0 \approx 20$°C;
Tp (t) - temperature in the fire chamber of the installation to determine the limits of fire resistance of structures depending on the time τ of the standard test.

The heat transfer coefficient takes into account the action of convection and infrared radiation and is determined by the formula:

$$\alpha = \alpha_B + \alpha_K \tag{4}$$

where: $\alpha_в$– heat transfer coefficient from radiation;
$\alpha_к$ - convection heat transfer coefficient.

According to the recommendations [8-10; 11] (IDT) convective and radiation components of the heating side can be determined from the expressions: α_K = 25 W / (m^2 K) - heated surface;

$$\alpha_B = \varepsilon \cdot \sigma \cdot \frac{T_W^4 - T_P^4}{T_W - T_P} \tag{5}$$

where: ε= 0,8 the degree of blackness of the wood surface;
σ = 5.67·10^{-8} W / (m^2 ºC^4) became Stefan-Boltzmann.

On the unheated surface of the beam, the expression GU III kind is written in the form:

$$-\lambda(T)\frac{\partial T}{\partial r}\bigg|_{r=h} = \alpha_n(293 - T_n), \tag{6}$$

where: α_n = 9 W / (m^2 K) - heat transfer coefficients of the unheated surface, which simultaneously takes into account its convective and radiation components.

The initial temperature of the beam material and the environment T_0 = 293 K.
Thermophysical characteristics of wood are accepted according to the recommendations of DSTU BN EN 1995-1-2: 2012 Eurocode 5 (IDT) [12].

The initial density of softwood according to [9; 11; 12] is ρ_0 = 520 kg / m^3.

The equation of nonstationary thermal conductivity (1) under these thermophysical characteristics, boundary conditions and this calculation area has no analytical solution. To solve this equation, one of the numerical methods must be used [13]. The method of finite differences is convenient for obtaining temperature fields for this mathematical model of thermal conductivity [13]. This method allows us to consider the equation of nonstationary thermal conductivity in linearized form.

The notation of the left-hand side of the thermal conductivity equation (1) in finite differences has the following form [12]:

$$A_{i,k} = Cv(T)\frac{\partial T}{\partial t} = Cv\left(\frac{T_{i,k} + T_{i,k+1}}{2}\right)\cdot\frac{T_{i,k+1} - T_{i,k}}{\Delta t}. \quad (7)$$

Partial derivatives in the right part of equation (1) with accuracy 0(h^2 + Δ*t*) are written in finite differences in the form of expressions [13]:

$$B_{i,k} = \frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) = a_x T^x_{i-1,k} - (a_x + b_x)T_{i,k} + b_x T^x_{i+1,k},$$
$$C_{i,k} = \frac{\partial}{\partial y}\left(\lambda(T)\frac{\partial T}{\partial y}\right) = a_y T^y_{i-1,k} - (a_y + b_y)T_{i,k} + b_y T^y_{i+1,k}, \quad (8)$$

The coefficients of equation (1) depending on the thermal conductivity are determined using the integra-interpolation method [13] by the following expressions:

$$\frac{1}{a} = h\int_0^{\Delta h}\frac{dh}{\lambda(T)} = \frac{(\lambda(T_{i-1}) + \lambda(T_i))h^2}{\lambda(T_{i-1})\lambda(T_i)}, \quad \frac{1}{b} = h\int_0^{\Delta h}\frac{dh}{\lambda(T)} = \frac{(\lambda(T_{i+1}) + \lambda(T_i))h^2}{\lambda(T_{i+1})\lambda(T_i)}. \quad (9)$$

Expressions (7) - (9) allow to approximate the equation of thermal conductivity by means of finite differences. In this case, the written equation is used in the form of recurrent formulas of the nodal points of the calculation region on the + 1st time layer at predetermined temperatures of the k-then time layer. Recurrent formulas are written implicitly and are solved as nonlinear equations.

The equations obtained by the finite difference method are written as:

$$A_{i,k} = B_{i,k} + C_{i,k}. \quad (10)$$

The solution of the obtained nonlinear equations is obtained by the method of half division, which is carried out according to the algorithm given in [13] by regularization by limiting possible solutions by the set of allowable heating temperatures of wooden beams.

Boundary conditions of the third kind in the final differences are written in the form [14]:

$$\frac{\lambda(Tw_k)\lambda(T_{1,k})}{\lambda(Tw_k)+\lambda(T_{1,k})}\cdot\frac{Tw_k-T_{1,k}}{h}+\frac{h\cdot Cv(T_{1,k})}{2\cdot\Delta t}\cdot(T_{1,k}-T_{1,k-1})=\alpha_k(T_{1,k}-T_{p,k}) \tag{11}$$

where T_p - temperature of the fire, which corresponds to the standard temperature regime and is determined by the formula of the standard temperature regime of the fire (3);

α– transfer coefficient determined by formulas (4) - (6);

h = 0.01 m - step of section division;

Δt = 60 c - time step.

The temporal and spatial step is chosen under the condition of convergence of the selected finite-difference scheme. Also, the time step is preferably selected equal to 1 minute, according to the control period of time when conducting fire tests.

To implement the method of finite differences, a finite-difference scheme was drawn up, shown in Fig. 1.

Interpolation of temperature distributions along cross-section lines. To approximate the temperature distributions along the cross-sectional lines of building structures when exposed to fire with a standard temperature, it is convenient to use the nomogram method [13]. In this case, the temperature distributions are represented by similar curves described by the functionals, where only their corresponding parameters differ. Analyzing the curves, we can assume that they can be approximated by this method [13].

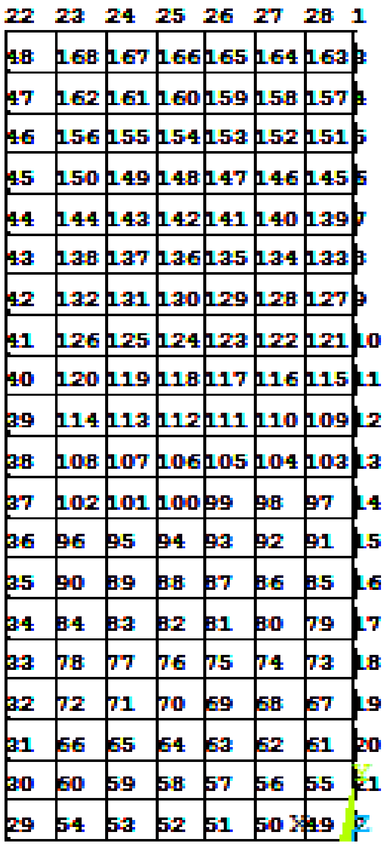

Fig. 2. Final-difference scheme of the investigated wooden beam.

To approximate them, we use an algorithm based on the representation of temperature curves by a generalized expression:

$$T_{k,i}=T_{0k}+(T_{\max k}-T_{0k})\left[\frac{i}{n}\right]^{Q_k}, \tag{12}$$

$T_{k,i}$ - the temperature of the i-th point of this section line at the k-th time;

T_{0k}, $T_{\max k}$ - temperature of the first and last points of this line of section at the k-th moment of time;

n - number of intervals between control points;

Qk - an indicator of the degree of the parabola at the k-th moment of time, which is determined by the algorithm presented in Fig. 3.

The results of the calculation for solving the thermal conductivity problem and the results are obtained using functionals of type (12).

The method described in [13] is used to interpolate the temperature fields at the intermediate nodal points of the cross section of the fragments of wooden beams under study. This method is based on approximate isotherms by special curves.

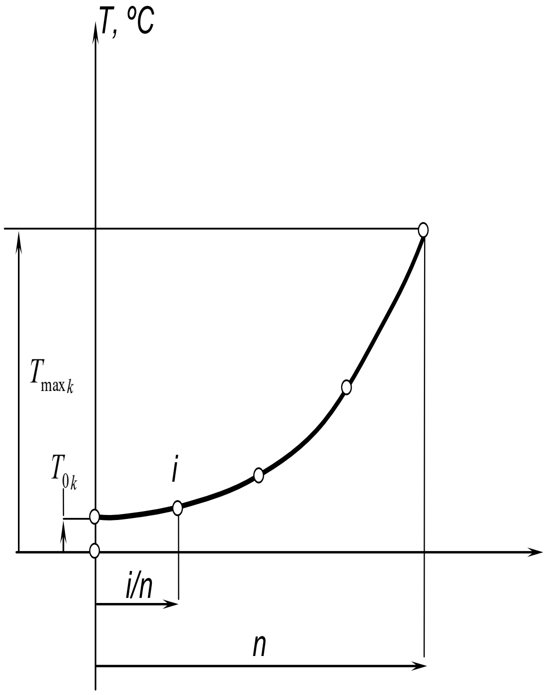

Fig. 3. The scheme of approximation of temperature distributions of a beam during tests on fire resistance.

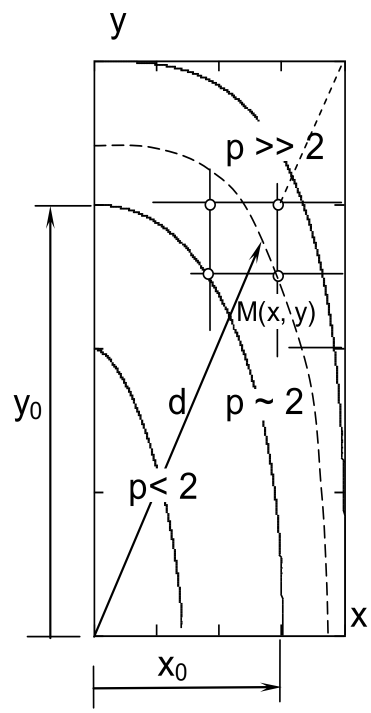

Fig. 4. Scheme of approximation of isotherms in the cross section of a fragment of a wooden beam.

Using this approach, a functionality having the following form was proposed:

$$y(x) = y_0 \left(1 - \left(\frac{x}{x_0} \right)^p \right)^{1/p} \qquad (13)$$

where x_0 and y_0 - coordinates on the x and y axes when they intersect with the approximating curve;

p - indicator of the degree of the approximating curve to be determined when approaching isotherms.

In parameterized form, expression (13) can be written as equation:

$$y^p x_0^p + x^p y_0^p - x_0^p y_0^p = 0, \qquad (14)$$

In this equation, each term depends on the parameter d, which determines the distance cut by the approximating curve on the main diagonal of the section. All members of the equation were determined by formulas.

$$y_0 = \left[(T_{dk} - T_0)(T_{vk} - T_0)^{-1} d^{Q_d}\right]^{Q_v^{-1}};$$

$$x_0 = \left[(T_{dk} - T_0)(T_{gk} - T_0)^{-1} d^{Q_d}\right]^{Q_g^{-1}}, \quad (15)$$

$$x = d \cdot a(a^2 + h^2); \quad y = d \cdot h(a^2 + h^2), \quad (16)$$

where *Tdk, Tgk, Tvk* - temperatures determined on the heating surfaces, respectively, on the diagonal and the average vertical and horizontal cross section;

T_0 - temperature of the least heated section point;

Qd, Qg, Qv - indicators of the degree of functional (14), which are defined respectively for the diagonal and the average horizontal and vertical section.

Varying the parameter d, the dependences of the exponent (16) for each minute during the action of a "standard" fire for 60 minutes were determined. Equation (16) was solved by the method of iterations according to the algorithm [14]. In fig. 3 shows graphs of the degree of dependence on the parameter d, for different points in the time of the fire.

Knowing the degree of functionality (15), it is easy to construct temperature distributions for a wooden beam. Using this approach and temperature measurements at the control points of the section during the tests, the interpolation of temperature distributions was performed.

Temperature dependences were used for interpolation. According to these dependences, it is possible to determine the curves of temperature distributions along the middle vertical section of the beam. In addition, the dependences of the index of the degree of functional (15) on the truncated segment of the diagonal of the symmetrical half of the cross section of the beam were used for interpolation. For interpolation, the dependences of the ratio of the coordinates of the intersection of the approximation curve $k_{xy} = \frac{x_0}{y_0}$ and the coordinate axes were also constructed. Knowing the listed data it is possible to construct curves which are approximation of isotherms in sections of the tested samples.

The cross section of the sample without fire protection heats up much faster. This is due to the fact that the impregnated layer not only resists combustion, but also reduces the thermal conductivity by sealing the surface layer.

When modeling the charring of wooden beams, we based the hypothesis that the charred part of the section should depend on its heating. If this is done, then the charring zone should be limited by the corresponding isotherm, which corresponds to the critical temperature at which 80% - 90% of the conversion of wood into coal.

Using regression dependences, it is possible to determine by the obtained approximation functional of type (14) for the middle horizontal section. The critical temperature of charring is determined by the formula:

$$T_{\text{кр},i} = T_{0i} + (T_{gi} - T_{0i})\left[\frac{0,5a - \beta \cdot t}{a}\right]^{Q_{gi}} \quad (17)$$

After determining the critical charring temperatures of wood, the charring zones were simulated for fragments of beams with and without fire-retardant impregnations, tested.

The study of the dynamics of charring of fragments of wooden beams shows that the two types at the beginning of the fire action of fire equally resist the spread of the charring zone, and this resistance is significantly greater than the resistance of the fragment without fire protection impregnation.

Thus, the temperature distributions in the cross section of the fragment of the wooden beam, which was tested using the recommendations containing the relevant standard for the calculation methods for assessing the fire resistance of wooden building structures, were determined.

The results of the calculation allow to develop an effective method of interpolation, to solve the strength problem by the proposed method, the calculation is performed according to the method described below.

1. The cross section of the beam is divided into rectangular zones, in each of which the temperature is determined as the average value of the temperatures of the four nodal points of the zone, known from the results of solving the thermal problem. The determined temperatures are recorded as matrices for each point in time.

2. Four concomitant matrices of coefficients of reduction of strength and modulus of tensile and compressive elasticity at each control time point are constructed.

3. At each current point in time, the functions of the dependence of stresses in this zone on the deformations are determined, according to which the deformation diagrams of the type given in Fig. 1 are constructed. 5, using expression (13).

4. Organizes a computational cycle, during which the step increases the curvature of the beam to the maximum value determined by formula (15). The vector of moments corresponding to the given current value of curvature at each control moment of time is constructed.

5. From the obtained values of the moment the largest value which is written down as the greatest effort sustained by a beam, ie its bearing capacity is chosen.

6. From the received values of bearing capacity of a beam for each current moment of time the corresponding vector is constructed.

7. Based on the obtained vector of bearing capacity values, a graph of its dependence on the time of fire is constructed.

8. The resulting graph can be used to determine the limit of fire resistance by comparing the value of load-bearing capacity and the load.

Thus, the operating load must be determined by the for

$$M_{Ed,fi} = 0.4M_{max}, \quad (18)$$

where M_{max} - maximum bearing capacity of the beam at normal temperatures.

Given the research, the following conclusions can be drawn.

- In this section, a method of interpolation of temperature fields in the cross section of fragments of wooden beams subjected to fire tests was developed.
- Using the developed method of interpolation, the temperature distributions in the cross section of the fragments of wooden beams are determined, taking into account the temperature measurements performed during the tests.
- Taking into account the obtained regularities of the rate of charring of wood fragments with and without fire protection, the critical charring temperatures were determined for each moment of the fire test time.
- Based on the obtained dependences, the charring zones of the fragments of wooden beams subjected to the test were determined.

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МЕТОДИ МАТЕМАТИЧНОГО МОДЕЛЮВАННЯ ЗОНИ ОБВУГЛЮВАННЯ ДЕРЕВ'ЯНИХ БАЛОК З ОБЛИЦЮВАННЯМ ВОГНЕЗАХИСНОЮ ФАНЕРОЮ

Аналіз існуючих методів математичного моделювання оцінювання вогнестійкості будівельних конструкцій залишаються актуальною темою для досліджень, оскільки, існує похибки досліджень, які повинні варіюватися з експериментальними дослідженнями. При цьому проведений аналіз дає можливість обрати більш досконалу інтегральну модель опису пожежі. За результатами проведеної роботи встановлено наступне:

Нами було проведено аналіз існуючих математичних методів моделювання процесів тепломасообміну при створенні теплових печей для визначення межі вогнестійкості дерев'яних балок з урахуванням можливих похибок дослідження.

На основі отриманих експериментальних даних за рахунок проведених вогневих випробувань зразків-фрагменті дерев'яних балок з облицюванням вогнезахисною фанерою.

На основі розв'язку диференціального рівняння нестаціонарної теплопровідності закону Фур'є та врахуванні методу кінцевих різниць, створена розрахункова модель для визначення зони обвуглювання дерев'яних балок з облицюванням вогнезахисною фанерою.

При цьому складність завдання в разі переходу до диференційованих зональних досліджень, які оптимізують розрахунок експериментальних даних для подальшого теоретичного узагальнення, не зменшується, оскільки зони процесу, розташовані в просторі, мають складний об'ємний характер навіть у найпростіших випадках факельного горіння.

Ключові слова: *вогнезахисна фанера, математичне моделювання, температурний режим пожежі, поширення пожежі, польові методи.*